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# ARITHMETIC

FOR

# "THE PREVIOUS,"

BEING EXPLANATIONS AND
HINTS ON THE HARDER RULES, AND EXAMPLES
FROM PAPERS,

BY

A. H. EVANS, B.A.,

*Late Scholar of Clare.*

CAMBRIDGE:
E. JOHNSON, TRINITY STREET.

1881.
*Price Two Shillings.*

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# PREFACE.

In the course of several years coaching, I have found the want of a book with examples, like those set in the "Little Go," worked out—and with one rule, as far as possible, to apply to all examples on the same subject. I hope that this attempt to meet the want may be found useful to those entering for Examination, as far as regards the latter part of the Arithmetic. No Problems are included, but only questions which may be worked by the Rules; and almost all the examples given are from "Little Go" papers.

A. H. EVANS.

16, Green Street,
*Nov. 10th*, 1881.

# CHAPTER I.

## PRACTICE.

THERE is no need to enter into the method of working examples in this rule at length; but a few points may be noticed, in which mistakes are often made.

(i.) When, in a division, the divisor does not go an exact number of times, and a fraction is therefore the result, *e.g.*

At any step in the Practice suppose we are dividing by 7,

| | £. | *s.* | *d.* |
|---|---|---|---|
| | 25 | 2 | 11 |
| $\frac{1}{7}$ | 3 | 11 | $10\frac{1}{7}$ |

Here is a remainder of '1' from the pence. In this case simply divide this remainder by putting it as numerator and your divisor as denominator, *i. e.* $\frac{1}{7}$.

Now suppose in the next step we are dividing by 5, *e. g.*

| | £. | *s.* | *d.* |
|---|---|---|---|
| | 25 | 2 | 11 |
| $\frac{1}{7}$ | 3 | 11 | $10\frac{1}{7}$ |
| $\frac{1}{5}$ | | 14 | $4\frac{3}{7}$ |

Here there is a remainder of '2' over from the pence, and also the fraction $\frac{1}{7}$.

*Rule* for this case. First add these together ($2\frac{1}{7} = \frac{15}{7}$), then divide by the 5, ($\frac{15}{7} \times \frac{1}{5} = \frac{3}{7}$).

If a fraction *only* remains to be divided, and no whole number: Simply divide this fraction by the divisor.

Never reduce lower than pence in any Practice sum.

(ii.) Find the value of 3 cwt. 2 qrs. 5 lbs. at £2. 3*s.* 6*d.* a quarter.

Here the price being given per "quarter," the hundredweights must be reduced to the same denomination, *i. e.* 3 cwt. 2 qrs. 5 lbs. = 14 qrs. 5 lbs., and then the example is worked in the usual way.

(iii.) Also we might have

3 cwt. 2 qrs. 5 lbs. at £2. 3*s*. 6*d*. *a ton.*

Here we work in the usual way; the only difference being that there are *no tons by which to muliply first, e.g.*

| | | £. | *s.* | *d.* |
|---|---|---|---|---|
| | | 2 | 3 | 6..... per 1 ton. |
| 2 cwt. | $\frac{1}{10}$ | | 4 | $4\frac{1}{5}$ |
| 1 cwt. | $\frac{1}{2}$ | | 2 | $2\frac{1}{10}$ |

and so on.

Two other remarks may be made:

(iv.) If you wish to find what part 2 lbs. is of 12 lbs. it is $\frac{2\text{ lbs.}}{12\text{ lbs.}} = \frac{1}{6}$,

or 2.oz of 12 lbs., it is $\frac{2.\text{oz}}{12\text{ lbs.}} = \frac{2.\text{oz}}{12 \times 16.\text{oz}} = \frac{1}{96}$: the rule being: Put the first for numerator and the second for denominator, reduce both *to the same denomination,* and then cancel.

(v.) When a case occurs such as 51143 things at £4. 17*s*. each:

| | | | |
|---|---|---|---|
| | | £51143 | |
| | | 4 | |
| | | 204572 | |
| 10 | $\frac{1}{2}$ | 25571 | 10*s*. |
| 5 | $\frac{1}{2}$ | 12785 | 15*s*. |
| 1 | $\frac{1}{5}$ | 2557 | 3*s*. |
| 1 | $\frac{1}{5}$ | 2557 | 3*s*. |

It is often much easier to take 1*s*. as $\frac{1}{5}$ of 5*s*. twice over than to say 2*s*. is $\frac{1}{5}$ of 10*s*.

## CHAPTER II.

## PROPORTION, OR RULE OF THREE.

Proportions would be always easy, if the following rule were strictly adhered to.

"First put down the third term, or term which has no other corresponding to it. Then take each pair of terms by itself *with reference to the last term,*" *e. g.*

(i.) "If 20 cannon, which fire 5 rounds in 6 minutes, kill 500 men in 1 hour: how many men will 10 cannon, firing 3 rounds in 5 minutes, kill in an hour and a half at the same rate?"

Here 500 men is the third term.

If 20 cannon in firing kill 500 men, how many men will 10 cannon kill—Less—*i.e.* Proportion is

$$20 : 10 :: 500 : \text{Ans.}$$

Next—If 5 rounds kill 500 men, how many men will 3 rounds kill—Less—so the terms stand 5 : 3.

Next—If 500 men are killed at a certain number of rounds in 6 minutes, how many will be killed if the same number of rounds are fired in 5 minutes—More—so we get 5 : 6.

Next—If 500 men are killed in one hour, how many will be killed in an hour and a half (= $\frac{3}{2}$ hours)—More—so, as before, 1 : $\frac{3}{2}$.

Putting all these down together

$$\begin{array}{rcl} 20 & : & 10 \\ 5 & : & 3 :: 500 : \text{Answer.} \\ 5 & : & 6 \\ 1 & : & \frac{3}{2} \end{array}$$

$$\text{Answer is } \frac{500 \times 10 \times 3 \times 6 \times 3}{20 \times 5 \times 5 \times 1 \times 2} = 270 \text{ men.}$$

(ii.) Notice cases such as this:

"If 5 men do a piece of work in 18 days, how many men will do a piece one-fifth of the size in 6 days?"

Here the ratio of the days is as before, 6 : 18, for it will take *more* men to do the work in *less* days.

The ratio of the work is of course 1 : $\frac{1}{5}$.

$$\therefore \text{Proportion is } \begin{array}{rcl} 6 & : & 18 :: 5 \text{ men} : \text{Answer.} \\ 1 & : & \frac{1}{5}. \end{array}$$

$$\text{and Answer} = \frac{5 \times 18 \times 1}{6 \times 1 \times 5} = 3 \text{ men.}$$

*N.B.*—It is always safest to put your answer down thus, as a fraction, before cancelling; remembering that dividing by a fraction is multiplying by it inverted, in cases where a fraction comes in the first term.

(iii.) Another kind of example often set is:

"If 3 men and four women can reap a field of $12\frac{1}{2}$ acres in $3\frac{1}{2}$ days, working 16 hours a day, in what time can 2 men and 10 women reap a field of 15 acres, working 12 hours a day, supposing one man's work is equivalent to that of two women.

Here we must reduce the men to women.

*i.e.*, 3 men and 4 women = 6 women + 4 women
= 10 women.

2 men and 10 women = 4 women + 10 women
= 14 women.

and the example then proceeds in the usual way.

## CHAPTER III.

## SIMPLE INTEREST.

Two rules apply to *all* cases of Simple Interest,

$$\text{viz., Interest} = \frac{\textit{Principal} \times \textit{Rate} \times \textit{Years.}}{100}$$

$$\text{Amount} = \frac{\textit{Principal} \times \textit{Rate} \times \textit{Years}}{100} + \text{Principal.}$$

The first finds the Simple Interest: and the second is merely the fact that the amount means the Principal plus the Interest, since the first term on the right-hand side is the Interest.

For any example merely fill these formulæ up, and work out the quantity required.

*Examples:*

(i.) What is the Simple Interest and Amount of £500., for 3 years, at 4 per cent.?

$$\text{Interest} = \frac{500 \times 3 \times 4}{100} = £60.$$

$$\text{Amount} = \frac{500 \times 3 \times 4}{100} + 500 = £560.$$

(ii.) On what sum will the Interest amount to £60. in 3 years, at 4 per cent.?

Here the Interest is £60. Principal is required:

$$60 = \frac{\text{Principal} \times 3 \times 4}{100}$$

$$6000 = \text{Principal} \times 3 \times 4.$$

$$\text{Principal} = \frac{6000}{3 \times 4} = £500.$$

(iii.) In what time will £500. produce £60., interest at 4 per cent.?

$$60 = \frac{500 \times 4 \times \text{years}}{100}.$$

$$6000 = 500 \times 4 \times \text{years}.$$

$$\text{years} = \frac{6000}{500 \times 4} = 3.$$

For Rate just the same method is used.

These examples (ii.) and (iii.) are cases where the Interest is given. If the *Amount* is given, we fill up the second of the two formulæ.

(iv.) What principal will amount to £560 in 3 years, at 4 per cent.?

Here amount = £560. Principal is required.

$$\therefore\ 560 = \frac{\text{Principal} \times 3 \times 4}{100} + \text{Principal}.$$

$$56000 = (\text{Principal} \times 3 \times 4) + 100\ \text{Principal}.$$

$$56000 = 112\ \text{Principal}.$$

$$\text{Principal} = \frac{56000}{112} = £500.$$

(v.) At what Rate will £500. amount to £560. in 3 years?

$$560 = \frac{500 \times 3 \times \text{Rate}}{100} + 500.$$

$$56000 = (500 \times 3 \times \text{Rate}) + 50000.$$

$$56000 - 50000 = 500 \times 3 \times \text{Rate}.$$

$$6000 = 500 \times 3 \times \text{Rate}.$$

$$\text{Rate} = \frac{6000}{500 \times 3} = 4 \text{ per cent.}$$

If time is required we work in exactly the same way.

(vi.) At what rate will a sum of money treble itself in 30 years. Here the amount = 3 Principal.

$$\therefore\ 3 \text{ Principal} = \frac{\text{Principal} \times 30 \times \text{Rate}}{100} + \text{Principal}.$$

$$3 = \frac{30 \times \text{Rate}}{100} + 1. \qquad \text{Rate} = 6\tfrac{2}{3}.$$

# CHAPTER IV.

## COMPOUND INTEREST.

THIS rule can be best seen by examples.

We take each year's interest *separately;* just as in Simple Interest; but at the end of each year we make up a fresh principal by adding the interest of that year to the principal of that year.

It is convenient to use Decimals in the working, for then we can divide by 100 by moving the decimal point two places to the left-hand.

(i.) What is the Compound Interest of £50. for 3 years at 5 per cent. ?

50 = First Principal.
5
250
1
250
Dividing by 100 = 2.50 = First Year's Interest.
52.50 = Second Principal (= 50 + 2.50.)
5
262.50
Dividing by 100 = 2.6250 = Second Year's Interest.
55.1250 = Third Principal = 52.50 + 262.50.
5
275.6250
Dividing by 100 = 2.756250 = Third Year's Interest.

The Compound Interest is therefore the sum of these three year's interests.

$$\begin{array}{r} 2.756250 \\ 2.6250\phantom{00} \\ 2.50\phantom{0000} \\ \hline £7.881250 \end{array}$$

$$\begin{array}{rr} = & £7.881250\phantom{0} \\ & 20 \\ \hline & 17.625000 \end{array}$$

$$\begin{array}{rr} = & £7.\ 17s.625 \\ & 12 \\ \hline & 7.500 \end{array}$$

$= £7.\ 17s.\ 7\frac{1}{2}d.$ (since $.500 = \frac{5}{10} = \frac{1}{2}$.

The Compound Amount would be £57. 17*s*. 7½*d.*

Now if the question had been for 3½ years, we should simply have taken the interest for the fourth year, and halved it; adding the result to our other three years.

But if the interest was to be reckoned ***half yearly***, we should work as before, with the exception that we should begin afresh every half-year instead of every year; for instance

(ii.) "Find the Compound Interest on £25000. at 5 per cent. for 2 years, interest being reckoned half-yearly."

Now for half-a-year's interest we shall multiply 25000 by 5 and by ½ and divide by 100.

$$\left.\begin{array}{l} \begin{array}{r} 25000 = \text{First Half-year's Principal.} \\ 5\phantom{= \text{First Half-year's Principal.}} \\ \hline 125000\phantom{= \text{First Half-year's Principal.}} \end{array} \\ \text{Half of this} = 62500. \\ \text{Dividing by } 100 = 625 = \text{First Half-year's Interest.} \end{array}\right\}$$

The sum of 25000 + 625 = 25625 = Second Half-year's Principal.

$$\begin{array}{r} 25625 \\ 5 \\ \hline 128125 \end{array}$$

Half of this = 64062.5.

Dividing by 100 = 640.625 = Second Half-year's Interest, and so on.

## CHAPTER V.

### DISCOUNT AND PRESENT WORTH.

The confusion which arises in the consideration of Discount is mainly due to the fact that, since the "Bill" is the main sum of money in an example, it is taken as the starting point for one's ideas.

But if instead of this we *first* look at the *Present Worth* the difficulty vanishes.

For begin with the Present Worth as a Principal. The Interest on it will be the "Discount," and the Bill will be the Amount, since it is the sum of the two former.

For example, suppose *A* and *B* are two men: and *A* has just sold the bill to *B*. Say it has 3 years to run.

*At the present time*
- *A* has Present Worth (*i.e.* what he gets for the Bill).
- *B* has Bill.

*In 3 years*
- *A* has Present worth *and* the Interest on it for 3 years.
- *B* has the Bill (which only now becomes money).

If the arrangement is fair, they must now have an equal sum.

Therefore Present Worth + Int. on Pres. Worth = Bill.

From this we see that the *Discount* is *Interest on Present Worth:* because we know that Present Worth and Discount = Bill.

*Examples :*

(i.) If Discount is given and Pres. Worth, required (or) Pres. Worth given and Discount required.

The one being the Interest on the other, these cases are Simple Interest sums, and not Discount at all, *e. g.*

1. If Discount is £20. for 4 years, at 5 per cent., what is Present Worth?

Here Interest is given = £20. ; find Principal. (*See* Ch. III. ii.).

$$20 = \frac{\text{Principal} \times 4 \times 5}{100}$$

$$2000 = \text{Principal} \times 4 \times 5$$

$$\text{Principal} = \frac{2000}{4 \times 5} = £100.$$

2. If Present Worth is £100. for 4 years, at 5 per cent., find the Discount.

$$\text{Here, Interest} = \frac{100 \times 4 \times 5}{100} = £20.$$

(ii.) When either the Discount (or Pres. Worth) is given and Bill required: or Bill is given and Discount (or Pres. Worth) required; we use the following formulæ.

(*a*) Amount of £100. : Bill : : Interest of £100. : Discount.
(*b*) Amount of £100. : Bill : : 100 : Pres. Worth.

N.B.—Since we saw above that the Bill was really an Amount, and the Discount was really an Interest, and the Present Worth was really a Principal; the formulæ are,

(*a*) Amount : Amount : : Interest : Interest.
(*b*) Amount : Amount : : Principal : Principal.

*Examples :*

1. Find the Discount on £929. 10*s.*, due 2¼ years hence, at 2½ per cent.

$$\text{Here, Interest of £100.} = \frac{100 \times 2\frac{1}{4} \times 2\frac{1}{2}}{100} = \frac{45}{8}£.$$

$$\text{Amount of £100.} = 100 + \frac{45}{8} = \frac{845}{8}£.$$

Now fill up the formula (*a*)

$$\frac{845}{8} : 929\tfrac{1}{2} :: \frac{45}{8} : \text{Discount};$$

$$\therefore \text{Discount} = \frac{45 \times 1859 \times 8}{8 \times 2 \times 845} = \frac{99}{2}$$

$$= £49.\ 10s.$$

2. On what Bill (or Sum) is £49. 10*s*. the Discount for 2¼ years, at 2½ per cent.

As before Amount of £100. $= \frac{845}{8}$£.

Int. of £100. $= \frac{45}{8}$£.

Fill up formula (*a*)

$$\therefore \frac{845}{8} : \text{Bill} :: \frac{45}{8} : 49\tfrac{1}{2}.$$

Multiplying extremes and means:

$$\frac{45}{8} \times \text{Bill} = \frac{845}{8} \times \frac{99}{2} = \frac{845 \times 99}{16};$$

$$2 \times 45 \times \text{Bill} = 845 \times 99;$$

$$\therefore \text{Bill} = \frac{845 \times 99}{2 \times 45} = \frac{1859}{2} = £929.\ 10s.$$

3. Where the discount is reckoned at *Compound* Interest, *e.g.*, "What is the discount on £286. 13*s*. payable in 2 years, reckoning Compound Interest at 5 per cent. per annum?"

This is exactly the same, as far as working out the formula goes, as the former examples; but the Interest and Amount of £100. must be compound.

Compound Interest of £100., for 2 years, at 5 per cent., is £10¼, amount is £110¼. Fill up the formula as before—

$$110\tfrac{1}{4} : 286\frac{13}{20} :: 10\tfrac{1}{4} : \text{Discount} \therefore \text{Discount} = £26.\ 13s.$$

(iii.) If the time is given thus, "A bill drawn, Aug. 9, for 8 months, is discounted on Oct. 17th."

Here the time is from the *discounting* to the end of the time the bill would run, *i.e.* from Oct. 17th to April 9th.

But if we have the days to count up in this way we add 3 days extra, called days of Grace, to the time.

(iv.) If we are asked for the *Mercantile Discount;* we have merely to find the Simple Interest on the bill; for that is what a tradesman would give, and not the Real or True Discount.

## CHAPTER VI.

## STOCKS.

The great difficulty experienced in examples on Stocks being, that each has to be worked in a somewhat different way; and that the method itself has to be first found: I think that the rule may be made much easier for men of ordinary ability by having *one* method for all examples, though it may appear rather Algebraic in some instances. Before proceeding to examples a few points may be noticed, in which mistakes are easily made.

1. If the stock is given in the form of shares, *e.g.*, 500 Bank Annuities, (or) 500 Gas Shares, at £5. each.

In the first instance not being informed how much the Annuities each are, we of course assume them to be £100., and therefore since there are 500 *Annuities* there will be £500,00 *Stock.*

In the second case there will be similarly £500. × 5 = £2500. Stock.

2. *Brokerage, e.g.* Let the Brokerage be ⅛ per cent.

The Rule is to join the Brokerage with the Price *before beginning* the example: and we only have to consider whether it is to be added or subtracted in each case.

Now if we are *buying* Stock, we *give* the Price and also *give* the Brokerage: so we add them. If we are *selling,* we *get* the Price but give the Brokerage: so we subtract.

Another catch is, if the Brokerage is given on the *actual* value, *e.g.*, "Invest at 120, the Brokerage being 1 per cent. on the actual value." Before we begin our example, we must find what the Brokerage is on 120 if it is 1 on 100—

$$100 : 120 :: 1 : x;$$

$$x = \frac{120 \times 1}{100} = \frac{6}{5} \text{ per cent.};$$

and then we work in the usual way.

Again, the Price is sometimes given thus, £144. per £200. bond. Here it will be £72. per £100. bond.

The Rule for working all Stocks is this,

(*A*) If Stock is what you wish to begin with,

$$\frac{\text{Stock}}{100} \times \text{Price} = \text{Money} \begin{cases} \text{obtained by selling (or) paid} \\ \text{when it was bought.} \end{cases}$$

(*B*) If money is what you begin with,

$$\frac{\text{Money}}{\text{Price}} \times 100 = \text{Stock}.$$

Notice that there is no fixed rule for *buying* or *selling;* but which formula to use simply depends on whether Stock or Money is taken to begin upon.

If you always work thus, whether the Stock or Money, *is given or not,* merely putting in the word "Stock," or "Money," or "Sum"; there will never be any difficulty. This will be clear from the following examples:

*Note.*—Be careful always that it is Stock you sell and Money with which you buy.

1. The 4 per cents. being at 82⅛, what must be given for £1000. Stock? and what sum would be gained by selling out again at 86¼?

Here £1000. *Stock* is given to begin upon;

$$\therefore \frac{1000}{100} \times 82\tfrac{1}{8} = \text{money (given for stock)};$$

$$= \frac{10 \times 657}{8} = \frac{3285}{4} = £821.\ 5s.$$

Secondly, we must sell out *Stock* of course;

$$\therefore \frac{1000}{100} \times 86\tfrac{1}{4} = \frac{10 \times 345}{4} = \frac{1725}{2} = £862.\ 10s.$$

Therefore gain = £862. 10*s*. − £821. 5*s*. = £41. 5*s*.

Notice that the "4 per cent." never comes in except to find the income, and *income* is always taken on *Stock*, being the Interest for one year.

(2). How much $3\frac{1}{2}$ per cent. Stock, at $97\frac{3}{4}$, can be purchased for £7038; and what income would be thence derived?

Here money is given—

$$\therefore \frac{7038 \times 100}{97\frac{3}{4}} = \text{Stock};$$

$$= \frac{7038 \times 4 \times 100}{891} = £7200.\ \text{Stock}.$$

Then the Income is $\frac{7200}{100} \times 3\frac{1}{2} = £252.$

(3). A person sells 16 Russian bonds of £100. each at $75\frac{7}{8}$, and invests the proceeds in a railway stock at 120: the brokerage for selling bonds is $\frac{1}{8}$ per cent. stock, and the brokerage, &c. on purchasing railway stock is 1 per cent. on the actual value. What amount of stock did he buy?

Here there are two catches:

Firstly, 16 bonds of £100. each = £1600. stock. Secondly, Brokerage in the second case is 1 per cent. on the actual value; and we must find what it is on 120. We found above in 2, (Brokerage) that it was $\frac{6}{5}$ per cent. on the stock.

Now sell £1600. *Stock* at $(75\frac{7}{8} - \frac{1}{8})$.

$$\frac{1600}{100} \times 75\tfrac{6}{8} = \textit{Money}$$

$$= 16 \times 75\tfrac{3}{4}.$$

Invest this again at $(120 + \frac{6}{5})$ and we shall get

$$\frac{16 \times 75\frac{3}{4} \times 100}{120\frac{6}{5}} = \textit{Stock.}$$

$$= \frac{16 \times 303 \times 5 \times 100}{4 \times 606} = \pounds 1000.\ \text{Stock.}$$

(4). £7200. 3½ per cent. stock is purchased for £7038. What is the price of stock?

Here both stock *and* money being given we may begin with either. Work as usual.

$$\frac{7200}{100} \times \text{Price} = \text{Money given} = 7038.$$

$$\therefore\ 7200 \cdot \text{Price} = 703800$$

$$\text{Price} = \frac{703800}{7200} = 97\tfrac{3}{4}.$$

(5). A person invests £3636. in railway shares paying 5½ per cent., the price of a £100. share being £120. He sells out when the shares are at 136, and invests the proceeds in Moscow Railway 5 per cent. debentures, the price being £101. per debenture of £100. Find the difference of the incomes derived from the two investments.

Here money is given, *i. e.* £3636.

$$\therefore\ \frac{3636}{120} \times 100 = \text{Stock (in first railway).}$$

He sells this stock at 136, *i. e.*

$$\frac{3636 \times 100}{120} \times \frac{136}{100} = \text{money obtained.}$$

He invests this money at 101 in the Moscow Railway,

$$\textit{i. e.}\ \frac{3636 \times 100 \times 136}{120 \times 100} \times \frac{100}{101} = \text{Stock.}$$

Now we know that income is always taken on *Stock;* and of course in cases like this, on the *first* and *last* Stock a person has.

$\therefore$ First income is on $\frac{3636}{120} \times 100$, and is

$$\frac{3636 \times 100}{120} \times \frac{5\frac{1}{2}}{100} \left\{ \begin{array}{l} \text{since the first per centage} \\ \text{is } 5\tfrac{1}{2}\text{;} \end{array} \right.$$

$$= \frac{3636}{120} \times \frac{11}{2} = \frac{3333}{20} = \text{£}166.\ 13s.$$

Second Income is on $\frac{3636 \times 100 \times 136 \times 100}{120 \times 100 \times 101}$, and is

$$\frac{3636 \times 100 \times 136 \times 100}{120 \times 100 \times 101} \times \frac{5}{100} \left\{ \begin{array}{l} \text{since second per centage} \\ \text{is 5 per cent.;} \end{array} \right.$$

$$= \frac{3636 \times 136 \times 5}{120 \times 101} = \text{£}204.$$

Therefore difference is £204. – £166. 13*s*.
= £37. 7*s*.

The next two examples are cases where we begin with "Stock," or "Money," where it is not given, merely using the word "Stock" or "Sum."

(6). A person holds a certain amount of Stock in the 3 per cents.; he sells out when they are at 96, and invests the proceeds in Railway Shares which are at £112. per £100. Share, and pay 5½ per cent. What amount of Stock in the 3 per cents. did he originally hold, if his Income from Railway Shares is £121?

$\frac{\text{Stock}}{100} \times 96 = \text{money}$; This *money* he invests at 112.

$\therefore \frac{\text{Stock}}{100} \times 96 \times \frac{100}{112} = \text{stock bought}$: Income on it is

$\frac{\text{Stock} \times 96 \times 100}{100 \times 112} \times \frac{5\frac{1}{2}}{100}$; and we are told this income is £121.

$$\therefore \frac{\text{Stock} \times 96 \times 100 \times 11}{100 \times 112 \times 200} = 121.$$

or $\frac{33\ \text{Stock}}{700} = 121$, or $33\ \text{Stock} = 700 \times 121$,

or $\text{Stock} = \frac{121 \times 700}{33} = \frac{7700}{3} = \text{£}2566.\ 13s.\ 4d.$

(7). Which is the more advantageous stock to invest in, the 3 per cents. at 94½ or the 2½ per cents. at 79?

Here we have to invest, but no sum is given,

$$\therefore \frac{\text{Sum} \times 100}{94\frac{1}{2}} = \text{Stock (in first Investment)}.$$

$$\text{and } \frac{\text{Sum} \times 100}{79} = \text{Stock (in second investment)}.$$

We require to find from which we obtain most Income.

$$\text{Now } \frac{\text{Sum} \times 100}{94\frac{1}{2}} \times \frac{3}{100} = \text{Income on First Stock.}$$

$$\text{and } \frac{\text{Sum} \times 100}{79} \times \frac{2\frac{1}{2}}{100} = \text{Income on Second Stock.}$$

These, when reduced, are equal to $\frac{2\text{ Sum}}{63}, \frac{5\text{ Sum}}{158}$.

Comparing these two fractions in the usual way by reducing to a common denominator, we get,

$$\frac{316\text{ Sum},\quad 315\text{ Sum}}{63 \times 158}$$

so that the first is the most advantageous.

Now if the question went on, "And what is the difference if a sum of £500. is invested," we have merely to put in £500. for Sum in the last step; *i. e.*

$$\frac{500 \times 316 - 315 \times 500}{63 \times 158}$$

and subtract the second from the first, we get

$$\frac{500\,(316 - 315)}{63 \times 158} = \frac{500}{63 \times 158}£ = \frac{250}{4977}£$$

$$= \frac{250 \times 20}{4977}\text{ shillings} = 1s.\ 0\tfrac{276}{4977}d.$$

(8). "A person holds stock to the amount of £2600. in the 5 per cents.; He sells out when they are at 104, and invests the proceeds in shares of a mine paying 6 per cent. What must be the price of the latter if his income from them exceed his former income by £8 13*s.* 4*d.*?"

Here we begin with £2600. Stock,

$$\therefore \frac{2600}{100} \times 104 = \text{money (obtained by sale)};$$

This he invests in the mine shares, of which we do not know the price,

$$\therefore \frac{2600 \times 104}{100} \times \frac{100}{\text{Price}} = \text{Stock (mining shares).}$$

Now the difference of Income is given. So we find the Incomes in the two cases.

On the last Stock the Income at 6 per cent. is

$$\frac{2600 \times 104 \times 100}{100 \text{ Price}} \times \frac{6}{100} = \frac{26 \times 104 \times 6}{\text{Price}} \pounds.$$

On the first Stock, *i. e.,* £2600., the income at 5 per cent. is

$$\frac{2600}{100} \times 5 = \pounds 130.$$

Now the first of these exceeds the last by £8⅔.,

$$i.\ e.\ \frac{26 \times 104 \times 6}{\text{Price}} - 130 = 8\tfrac{2}{3}.$$

$$\frac{26 \times 104 \times 6}{\text{Price}} = 138\tfrac{2}{3} = \frac{416}{3}$$

$$\therefore 416 \text{ Price} = 3 \times 26 \times 104 \times 6. \quad \text{Price} = 117.$$

# CHAPTER VII.

## PROFIT AND LOSS.

There are two varieties of this rule:

(i.) Given the cost and selling price, find the gain per cent.

(ii.) Given the selling (or cost) price and gain (or loss) per cent. find the cost (or selling) price.

The first kind is very easy, *e.g.,*

1. If Sugar is bought for 7*s.* 6*d.* a lb. and sold for 8*s.* 3*d.*, what is the gain per cent.?

Now the gain is always on the *cost* price. The difference here is 9*d.*, which is the gain on 7*s.* 6*d.*—so, if 9*d.* is the gain on 7*s.* 6*d.*, what is the gain on 100?

$$7s.\ 6d. : 100 :: 9d. : \text{Ans.}$$

$$\text{Answer} = \frac{100 \times 9}{90} = 10 \text{ per cent.}$$

Now if the question had stood thus, "If Sugar is bought for 7*s*. 6*d*. a lb., and sold for 8*s*. 3*d*., what is the gain on 3 qrs. 1 lb.?" We should have worked thus: The gain is 9*d*. on 1 lb. (instead of on 7*s*. 6*d*.). what is it on 3 qrs. 1 lb. (= 85 lbs.),

$$\therefore 1 : 85 :: 9 : \text{Answer.}$$

$$\text{Answer} = 9 \times 85d. = £3.\ 3s.\ 9d.$$

2. The second kind consists of examples such as this: "By selling Tobacco at 1*s*. 3*d*. a lb. a Tobacconist gains 35 per cent. What was the cost price per lb.?" For 100 he gets 135; If for what he gave 100 he gets 135, what did he give for 1*s*. 3*d*.?

$$135 : 1s.\ 3d. :: 100 : \text{Answer.}$$

$$\text{Answer} = \frac{100 \times 15d.}{135} = \frac{100d.}{9} = 11\tfrac{1}{9}d.$$

Notice that in these sort of examples the proportion either runs:

$$\text{cost price of 100} : \text{cost price given} :: \left.\begin{matrix}\text{selling price}\\ \text{of 100}\end{matrix}\right\} : \text{Answer.}$$

$$\text{or,}\quad \left.\begin{matrix}\text{selling price}\\ \text{of 100}\end{matrix}\right\} : \left.\begin{matrix}\text{selling price}\\ \text{given}\end{matrix}\right\} :: \left.\begin{matrix}\text{cost price}\\ \text{of 100}\end{matrix}\right\} : \text{Answer.}$$

3. "A boy buys a certain number of apples at 2 for 1*d*., and half that number at 4 for 3*d*. At what price must he sell so as to gain 20 per cent. on his outlay?"

This is really the same as the last example. But one difficulty comes in, namely, we must find the cost price first, before we can state our proportion.

The method is this. Take what we may call one "set" of apples, *i. e.*, in this case,

$$\left.\begin{matrix}\text{2 of first sort}\\ \text{1 of second sort}\end{matrix}\right\}\text{since he buys in the ratio } 2 : 1.$$

Now the cost of these 3 apples is,

$$\left.\begin{matrix}1d.\ \text{for 2 of the first sort}\\ \tfrac{3}{4}d.\ \text{for 1 of the second sort}\end{matrix}\right\} = 1\tfrac{3}{4}d.$$

and since he gains 20 per cent., he gets 120 for 100. What then does he get for $1\frac{3}{4}d.$?

$$100 : 1\tfrac{3}{4}d. :: 120 : \text{Answer.}$$

$$\text{Answer} = \frac{120 \times 7}{100 \times 4} = \frac{21d.}{10}.$$

Now this is the price of our 'set' of 3 apples.

$$\text{So 3 apples cost } \frac{21}{10}$$

$$\text{1 apple costs } \frac{1}{3} \times \frac{21}{10} = \frac{7d.}{10}$$

$$\therefore \text{ 10 apples cost } 7d.$$

or 7*d.* for 10 apples is the price.

4. Another example of the same sort, involving both the methods (i.) and (ii.) is this—

"A grocer has a stock of tea which he must sell at 3*s.* $4\frac{1}{2}d.$ a lb. in order to gain 35 per cent. How much will he gain per cent. if he mixes it in the proportion of 5 : 2 with tea which cost him 2*s.* 1*d.* per lb., and sells the mixture at the rate of 14 lbs. for £2. 5*s.*?"

Here we first find the cost price per lb., before we can make our mixture, as in Example 3.

First Process. He gets 135 for what he gave 100. What did he give for 3*s.* $4\frac{1}{2}d.$

$$135 : 3s.\ 4\tfrac{1}{2}d. :: 100 : \text{Answer.}$$

$$\text{Answer} = \frac{100 \times 81}{135 \times 2} = 30d. = \text{cost per lb.}$$

Second Process. Take one set of lbs. which will be 5 of one kind, 2 of the other = 7 lbs.

$$\begin{aligned}\text{The cost of this is } & (5 \times 30d.) + (2 \times 25d.) \\ &= 200d. \text{ for 7 lbs.} \\ &= \tfrac{1}{7} \text{ of } 200d. \text{ for 1 lb.} \\ &= 28\tfrac{4}{7} \text{ per 1 lb.}\end{aligned}$$

The selling price will be $\frac{1}{14}$ of £2. 5*s.* per lb.
since 14 lbs. of the mixture costs £2. 5*s.*
It will be therefore $38\frac{4}{7}$ per lb.
$\therefore$ gain per lb. will be $38\frac{4}{7}d. - 28\frac{4}{7}d. = 10d.$
If 10 is gain on $28\frac{4}{7}$ what will it be per cent.?

$$28\tfrac{4}{7} : 100 :: 10d. : \text{answer.}$$

$$\text{Final Answer} = \frac{10 \times 100 \times 7}{200} = 35 \text{ per cent.}$$

5. Another very similar example is—

"A wine merchant buys whisky at 17*s*. 6*d*. per gallon, and after adding water to it in the proportion of 1 gallon to 6, retails it at 18*s*. per gallon. What percentage of profit does he make on his outlay?"

Here one "set" of gallons is 7.

The cost of 6 gallons of whisky at 17*s*. 6*d*. is

$$6 \times 17\tfrac{1}{2}s. = 6 \times \tfrac{35}{2} = 105s.$$

and the water costs nothing, so this is the cost price of the 7 gallons.

The selling price is 7 gallons at 18*s*. = 126*s*.

The gain is therefore 126*s*. — 105*s*. = 21*s*.

If 21*s*. is the gain on 105*s*., what is the gain on 100?

$$105 : 100 :: 21 : \text{Answer.}$$

$$\text{Answer} = \frac{100 \times 21}{105} = 20 \text{ per cent.}$$

6. But if the proportion of one ingredient to the other is required, the matter is not quite so easy. "A wine merchant buys gin at 12*s*. 6*d*. a gallon, and after adding water retails it at 13*s*. 4*d*. He makes 20 per cent. profit on his outlay; what proportion of water does he add to the gin?

Here 13*s*. 4*d*. is the selling price of *mixture*; what would cost be, if he gains 20 per cent.?

As in example 2. 120 : 100 :: 13*s*. 4*d*. : Answer.

$$\text{This Answer} = \frac{400d.}{3} = \frac{100}{9} \text{ Shillings per gall.}$$

*The rule is this*—subtract this cost of the mixture from the original cost, and the ratio of the resulting quantity to the cost of mixture will be the required ratio of water to gin, *i.e.*,

$$12\tfrac{1}{2}s. - \frac{100s.}{9} = \frac{25s.}{18}. \quad \text{Therefore the ratio is } \frac{25}{18} : \frac{100}{9},$$

$$\text{or } \frac{25}{18} \times \frac{9}{100} = \frac{1}{8} \left(\text{since the ratio } a : b \text{ is } \frac{a}{b}\right);$$

∴ Answer is 1 gallon of water to 8 of gin.

7. "A grocer has two sorts of tea which cost 3*s.* and 3*s.* 6*d.* a lb. respectively; in what ratio must he mix them so as to gain 10 per cent. by selling the mixture at 3*s.* 8*d.* a lb.?

Find cost of mixture as before. It is 40*d.*

*The rule is this*—Subtract it from the 3*s.* 6*d.* and also the 3*s.* from it, the ratio of these two results will be the required ratio, *e.g.*,

42*d.* − 40*d.* = 2*d.* and 40*d.* − 36*d.* = 4*d.*; so the ratio is 2 : 4 or 1 : 2 $\left\{\begin{array}{l}\textit{i. e.}\text{, 1 lb. of the first and 2 of the}\\ \text{second kind.}\end{array}\right.$

## CHAPTER VIII.

## PROPORTIONAL PARTS AND PARTNERSHIPS.

(*A*). *Proportional Parts.*

1. If merely asked to divide £360. into parts in the ratio of 2 : 3 : 4 (or what is the same thing, between *A*, *B* and *C* so that *A* may have half of what *C* has, and *B* half as much as *A* and *C* together) merely add the numbers 2, 3, 4 (= 9), and put each of them on the 9, *e.g.*,

$\frac{2}{9}, \frac{3}{9}, \frac{4}{9}$ $\left\{\begin{array}{l}\text{These will be the required parts}\\ \text{of £360.}\end{array}\right.$

*A* will have $\frac{2}{9} \times 360 =$ £80.
*B* ,, ,, $\frac{3}{9} \times 360 =$ £120.
*C* ,, ,, $\frac{4}{9} \times 360 =$ £160.

The rule is worked in exactly the same way if the ratios are fractions, *e.g.*, $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$.

The parts would be $\frac{\frac{1}{2}}{\frac{13}{12}}, \frac{\frac{1}{3}}{\frac{13}{12}}, \frac{\frac{1}{4}}{\frac{13}{12}}$;

or $\frac{6}{13}, \frac{4}{13}, \frac{3}{13}$.

2. This example is worked in exactly the same way. "Four towns contribute to the Bengal Famine Fund in proportion to their population. Their contributions are respectively £151. 17*s.* 6*d.*, £272. 16*s.* 3*d.*, £360. and £612. 11*s.* 3*d.* The total population of the four towns is 62100. What is the population of each?"

The meaning of this is, "Divide 62100 in the ratio of £151$\frac{7}{8}$. : £272$\frac{13}{16}$. : £360. : £612$\frac{9}{16}$. The sum of these is £1397$\frac{1}{4}$.

So the part corresponding to the first town is

$$\left.\frac{151\frac{7}{8}}{1397\frac{1}{4}} = \frac{5}{46}\right\} \begin{array}{l}\text{Therefore its population}\\ \text{is } \frac{5}{46} \times 62100 = 6750.\end{array}$$

and the others come in the same way.

3. "If a compound consists of two ingredients in the ratio of 2 : 3, find how much of each there is in 2 cwt. 3 qrs. 10 lb. of the mixture." This is worked in exactly the same way.

(*B*). *Partnerships.* These are simply Proportional Parts again.

1. "If a man enters into Partnership with another, the first with £200., and the second with £300., and their profits at the year's end are £600., what part should each get?"

This, as before, is simply "Divide £600. in the ratio of 200 : 300." Now we may reduce this ratio, *i. e.*, 200 : 300 will become 2 : 3. The parts will be

$$\left.\begin{array}{l}\frac{2}{5}\text{ of £600.}\\ \frac{3}{5}\text{ of £600}\end{array}\right\} \text{ or } \left.\begin{array}{l}\text{£240.}\\ \text{£360.}\end{array}\right\} \text{ Answer.}$$

Notice that here the "times" are the same, but in the next example, they are different. The rule is, in the second case, to multiply each man's money by the time he is in Partnership, and then work as usual.

2. "*A* invests £200. in a business; and after 3 months *B* comes in with a capital of £300. What proportion of the profits will each get at the end of the year, if they make £340.?"

$$\text{Here } A \left.\begin{array}{r}\text{has £200.}\\ \text{for 12 months}\end{array}\right\} \quad B \left.\begin{array}{l}\text{has £300.}\\ \text{for 9 months}\end{array}\right\}$$

By the above rule 2400 : 2700 is the ratio;
or 8 : 9.

The parts will then be $\frac{8}{17}$ of £340. and $\frac{9}{17}$ of £340.; or *A's* share = £160. *B's* share = £180.

# CHAPTER IX.

## SQUARE ROOT.

I shall only give under this head the rule for square roots of decimal quantities, and shall not enter into the well known method for whole numbers.

Required the Square Root of 124.3225.

Now the rule for pointing is not, as in whole numbers, "Begin on the last figure of the quantity and point every second figure to the left hand," but is, "Begin on the last figure of the *whole number*, and point every second figure as usual, both towards the *right hand* and the *left*.

Then work as usual, *e. g.*,

$$\begin{array}{r|l}
 & \dot{1}2\dot{4}.3\dot{2}2\dot{5} \;\big|\; \underline{11.15} \\
 & \underline{1} \\
\underline{21} & 24 \\
 & \underline{21} \\
\underline{221} & 332 \\
 & \underline{221} \\
\underline{2225} & 11125 \\
 & \underline{11125}
\end{array}$$

*Note* 1.—The decimal point comes into the answer as soon as you take in the first figure past the decimal point in the given quantity.

*Note* 2. Of course as many "noughts" as you like can be added at the end of a decimal.

The only other examples necessary are the cases where the decimal is a recurrer.

Here we turn them into fractions, *e. g.*

(i.) Required the square root of $.\dot{4}$ $(=\frac{4}{9})$

Root is $\frac{2}{3}$ or as a decimal again $.\dot{6}$.

(ii.) Required the square root of $1.36\dot{1}$.

$$\text{Now } 1.36\dot{1} = 1\tfrac{325}{900} = \tfrac{1225}{900},$$

$$\text{So the root is } \frac{35}{30} = \frac{7}{6} = 1.1\dot{6}.$$

Questions are not uncommon which involve square root, *i.e.* "Find the value of $\sqrt{.121} - .04\dot{7}$ to 3 places of decimals.

Here $\sqrt{.121}$ must be found first.

$$\begin{array}{r|l} & .\dot{1}2\dot{1}0\dot{0}0\ldots\ldots \quad \underline{|\ .347\ldots\ldots} \\ & 9 \\ \underline{64} & \overline{310} \\ & 256 \\ \underline{687} & \overline{5400} \\ & \underline{4809} \end{array}$$

and so on.

$$\text{Therefore } \sqrt{.121} - .04\dot{7} = \frac{347}{1000} - \frac{43}{900}$$

$$= \frac{3123 - 430}{9000} = \frac{2693}{9000} = .299\dot{2}.$$

Correct to 3 places it is .299.

## CHAPTER X.

### SURFACES AND CUBIC CONTENTS.

(*A*). *Surfaces.*

The rule for finding the surface of a floor, or of the walls of a room, are as follows:

(1). For a floor, ceiling or any square area. Multiply the length by the breadth, after reducing both quantities to the *same denomination.*

Also notice that since L. × B. = Area, either $\text{L.} = \frac{\text{Area}}{\text{B.}}$ or $\text{B.} = \frac{\text{Area}}{\text{L.}}$, *i. e.*, if we are given Area and Breadth we divide to get the Length; and if we are given Area and Length we divide to get the Breadth.

(ii). For the walls of a room. Take the area of each wall separately as in (1). Here, the area of each of the longest walls will be Length × Height, and of the shortest walls Breadth × Height of Room. The whole area is the sum of these two, *i. e.*,

$$2L. \times H. + 2B. \times H.;$$
$$\text{or } 2H. (L. + B.)$$

So we might have first added the Length and Breadth of the Room, if we preferred, and then multiplied by twice the Height. From the Area already found, we must subtract areas of any windows, doors, etc., each found separately. This will give our final area.

*Examples:*

1. Find the cost of paving a floor 11 yards 2 feet long, by 9 yards $2\frac{2}{5}$ feet wide, with tiles 7 inches square, which cost £3. 4*s*. 7*d*. per 100.

Here, Area of floor $= 35 \text{ feet} \times 29\frac{2}{5} \text{ feet}$

$$= \frac{35 \times 147}{5} \text{ square feet}$$

$$= \frac{35 \times 147}{5} \times 144 \text{ square inches.}$$

We reduce the Area to inches because the tiles by which we are going to divide are in inches.

Now each tile $= 7 \times 7 = 49$ square inches, and if we divide the size of each tile into the whole area it will give us the number of tiles;

$$\therefore \text{ No. of Tiles} = \frac{35 \times 147 \times 144}{5 \times 49} = (21 \times 144).$$

We then get a Proportion. If 100 tiles cost £3. 4*s*. 7*d*., what will (21 × 144) cost?

$$100 : 21 \times 144 :: £3.\ 4s.\ 7d. : \text{Answer.}$$

$$\text{Answer} = \frac{775 \times 21 \times 144}{100} \text{ pence}$$

$$= \frac{31 \times 21 \times 36}{20 \times 12} £.$$

$$= £97.\ 13s.$$

2. "In a turnip field the rows are 2 feet apart and the turnips 9 inches apart in the rows: if the weight of the turnips average 1 lb. 1oz. a piece, what will be the weight of the whole crop on an acre?

The chief thing to be noticed here is that the field being an acre is *a square*; for an acre is square measure.

So the area instead of being Length × Breadth, is Length × Length or $(\text{Length})^2$ and this is of course equal to the acre.

Therefore $(\text{Length})^2 = 1$ acre $= 4840$ yards (square).

Taking the square root, Length $= \sqrt{4840}$, or,

Length $= \sqrt{484} \times \sqrt{10} = 22\sqrt{10}$ yards.

Now there are two feet between the rows, so the number of rows will be found by dividing the length of a side by the two feet; taking care that both are in the same denomination, *i. e.* No. of Rows $= \dfrac{22\sqrt{10} \text{ yards}}{2 \text{ feet}} = \dfrac{3 \times 22\sqrt{10} \text{ feet}}{2 \text{ feet}}$

$= 33\sqrt{10}$ feet.

Now take the other side which is also equal to $22\sqrt{10}$. If we divide this by the distance between the turnips we shall get the number of turnips in a row; in exactly the same way we found the number of rows, *i. e.*

$$\frac{22\sqrt{10} \text{ yards}}{9 \text{ inches}} = \frac{3 \times 12 \times 22\sqrt{10} \text{ inches}}{9 \text{ inches}}$$

$= 88\sqrt{10}$ turnips in a row.

Now there are $33\sqrt{10}$ rows and $88\sqrt{10}$ turnips in each, the whole number of turnips will be

$33\sqrt{10} \times 88\sqrt{10} = 33 \times 88 \times 10$ (since $\sqrt{10} \times \sqrt{10} = 10$) each of these weigh 1lb. 1oz. = 17 oz.

Therefore the whole weights will be $(33 \times 88 \times 10) \times 17$ oz.

Work this out and reduce to lbs. quarters, &c. It will be found to be 13 tons 15 cwt. 1 qr. 27 lbs.

3. A square Lawn is bordered by a path 4 feet 6 inches wide—the path and lawn together being equal to one tenth of an acre. Find the cost of paving the path with bricks at 7*s*. 6*d*. per square yard.

Here the difficulty is just the same as in the last example. The Area is square and is equal to $\frac{1}{10}$ acre
$= \frac{1}{10} \times 4840$ yards (square)
$= 484$ square yards.

As before the square root of this will be the length of each side of the square composed of the lawn and path together, and will be equal to 22 yards.

The path may be considered as double the sides plus double the ends.

Now the area of each side part will be its length multiplied by its breadth = 22 yards × 4 feet 6 inches = 22 × 3 feet × $4\frac{1}{2}$ feet = 33 × 9 square feet.

The area of each end part will be its length multiplied by its breadth also,

But the length *will not be 22 yards* (or we should take in the corners again, which we have already taken). If you draw the figure this will easily be seen. The Length will be, 22 yards − 2 (4 feet 6 inches) = 22 × 3 feet − 9 feet = 57 feet.

Notice this carefully as it is a catch which always comes in questions of paths round fields, &c.

So the area of each end will be 57 feet × $\frac{9}{2}$ feet $= \frac{57 \times 9}{2}$ square feet. Double this and also the area of the side, (since there are two ends and two sides), and add them. We get

$$2 \times 33 \times 9 + 2 \times \frac{57 \times 9}{2} \text{ square feet}$$
$$= 1107 \text{ square feet} = 123 \text{ square yards.}$$

We reduce to yards because the price is given per square yard. Each of these costs 7*s.* 6*d.* So the whole cost will be 123 × $7\frac{1}{2}$ shillings

$$= \frac{123 \times 15}{2} \text{ shillings} = \frac{1845£}{2 \times 20} = £46.\ 2s.\ 6d.$$

(*B*). *Cubic Content.* The rule for finding a cubic content is to multiply all the three dimensions together, *e.g.*, Length × Breadth × Depth, taking care they are all in the same denomination.

1. "Find the number of cubic feet in a cistern of water (or 'in a log of wood' etc., for it is really the same thing) 6 feet by 4 feet, if the water is 1 yard deep."

Cubic Content is merely 6 feet × 4 feet × 1 yard, or 6 feet × 4 feet × 3 feet = 72 cubic feet.

2. The edges of a brick are $6\frac{3}{4}$, $4\frac{1}{2}$, $2\frac{2}{5}$ inches respectively, how many can be stacked in a space equal to a cubic yard?"

Here cubic content of one brick $= 6\frac{3}{4} \times 4\frac{1}{2} \times 2\frac{2}{5}$

$$= \frac{27 \times 9 \times 12}{4 \times 2 \times 5} \text{ cubic inches.}$$

Now a cubic yard is equal to 1728 × 27 cub. in., and the number of times $\frac{27 \times 9 \times 12}{4 \times 2 \times 5}$ will go into this will be the number of bricks that will go into the yard.

$$\therefore \text{No. of Bricks} = 1728 \times 27 \times \frac{4 \times 2 \times 5}{27 \times 9 \times 12} = 640.$$

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# CHAPTER XI.

*Chain Rule: for Exchanges, etc.*, is a very useful rule for examples such as the following.

1. "If 5 fowls are worth 3 ducks, 14 ducks worth 5 geese, and 3 geese worth 2 turkeys, what is the price of a fowl when a turkey costs a guinea.

Put them down in pairs thus:

5 fowls = 3 ducks.
14 ducks = 5 geese.
3 geese = 2 turkeys.
1 turkey = 1 guinea.
Required price of fowl = 1 fowl (in guineas).

*Note.*—The rule is that you must arrange them so that the last of every pair is the same sort as the first of the next pair, and so that the end term of the whole is the same as the first of all.

Multiply all on each side together.

$$\therefore 5 \times 14 \times 3 \times 1 \times (\text{Price of Fowl}) = 3 \times 5 \times 2 \times 1 \times 1$$

$$\therefore \text{Price of fowl} = \frac{3 \times 5 \times 2 \times 1 \times 1}{5 \times 14 \times 3 \times 1} \text{ guineas.}$$

$$= \frac{1}{7} \text{guineas} = 3 \text{ shillings.}$$

2. If 20 francs be equal to 22.5 lire in Italy, and £5. sterling be worth 127 francs, express £24. 16*s*. 8*d*. in lire and centesimi (1 lira = 100 centesimi). Here our terms are not in order, and want arranging.

$$\left.\begin{array}{r} \pounds 5. = 127 \text{ francs.} \\ 20 \text{ francs} = 22.5 \text{ lire.} \\ \text{Required Lire} = 24\tfrac{5}{6}\pounds. \end{array}\right\}$$

$$\therefore 5 \times 20 \times \text{Lire} = 127 \times 22\tfrac{1}{2} \times 24\tfrac{5}{6};$$

$$\therefore \text{Lire} = \frac{127 \times 22\tfrac{1}{2} \times 24\tfrac{5}{6}}{5 \times 20}$$

$$= \frac{56769}{80} = 709\tfrac{49}{80} \text{ Lire}$$

$$= 709 \text{ Lire, } \frac{49}{80} \times 100 \text{ Centesimi}$$

709 Lire. 61¼ Centesimi.

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